Properties of Spherical Lens Antennas for High Altitude Platform Communications.
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Abstract. HAPs may exploit millimetre-wave spectrum by supporting multi-cell architectures. The overall data capacity is governed by the properties of the antennas which serve the cells. Optimum coverage is expected when each antenna radiation pattern is tailored to its respective cell, but this leads to a bulky payload. In contrast, this paper shows how a payload based on multi-beam spherical lenses, which have wide-scan properties but also offer circular beams, can give comparable performance while offering a much more compact payload. Models for spherical lens radiation patterns are presented which include the effect of the primary feed beamwidth. This allows a comparison of aperture diameter, sidelobe level, and the resulting co-channel interference, from which may be estimated the performance and relative mass of different antenna payload types.

INTRODUCTION.
This paper concerns communications from High Altitude Platforms (HAPs), which may be based on either aircraft or balloons, and stationed in the stratosphere at altitudes of around 17-22 km [1], [2]. This geometry will allow for line-of-sight links which are required for effective exploitation of the high bandwidths available at mm-wave frequencies. Multi-cell architectures will allow for spectrum re-use and will require a multi-beam HAP payload [4] possibly using a group of spot-beam antennas. The level of carrier-to-interference ratio (CIR) is a consequence of the spatial overlap of the antenna radiation patterns.

The ITU have allocated the 47/48 GHz band [6] for usage by HAPs worldwide, and the 27/31 GHz band has also been allocated to about 40 countries [7]. At these frequencies it is difficult and expensive to produce electronically re-configurable antennas based on phased-array principles and digital beam forming (DBF). For example, DBF presents challenges associated with: the close spacing of radiating elements, the requirement for a large number of elements, the need to minimise sidelobe levels, the cost and complexity of RF active circuits, and the problem of scanning loss. The latter point is particularly detrimental for HAPs communications because links to users at low elevation angles, where scanning loss for a horizontal planar array are worst, experience also the longest free space and atmospheric losses. Nevertheless a number of programmes are developing DBF antennas for Ka band e.g. [8] which reports on a 16-element module, and the Japanese HAPs programme [9] have reported a similar prototype for 28 GHz while conceding that horn antennas stabilised by mechanical gimbals are a more pragmatic solution for 48 GHz.

The more conservative solution for the HAP antenna payload is a group of aperture antennas, where each antenna serves a single quasi-stationary cell. This approach has been studied in some depth [4], [5], [10] where the important relationship between the radiation patterns and the CIR levels on the ground has been quantified for various channel re-use schemes. Ultimately, the economic viability of a HAP cellular network may be determined by the extent to which it can maximise the re-use of spectrum. This paper explores the properties of multi-beam antennas based on spherical lenses, since these can yield a much more compact HAP antenna payload compared to the use of single-beam aperture antennas. An estimate of payload mass compared to CIR coverage levels is introduced.

DEDICATED BEAMS FOR THE HAP ANTENNA PAYLOAD.
It is worth listing the pros and cons of using dedicated beams, i.e. one antenna per cell. Advantages include:
- each antenna pattern can be tailored to the shape required by each cell.
- asymmetric beams yield equal size, circular cells, which aids tessellation of cells and maximises coverage [11].
- low sidelobe levels are feasible e.g. -40 dB relative to peak gain.

Disadvantages include:
- A bulky, massive payload, which requires mechanical stabilisation.
- A non-reconfigurable network.

These disadvantages can to some extent be overcome with multiple-beam antennas which allow for the payload mass and volume to be reduced, but at the expense of eroding the advantages associated with dedicated beams because it becomes more difficult to tailor each beam to each cell. Multi-beam aperture antennas typically use a cluster of primary feeds to illuminate a secondary aperture which might be a prime-focus reflector or lens, but the angular range of scanning is limited. For a HAP payload a wide conical scan is needed e.g. 150° for users at elevation angles as low as
The following sections explore the potential to use multiple-beam, wide-scanning antennas based on spherical lenses, and quantify the CIR values which could be expected.

**PROPERTIES OF SPHERICAL LENS ANTENNAS.**

In the context of HAPs communications, the very useful property of antennas using spherical lenses is their ability to form multiple beams over a wide range of angles without inducing any scanning loss. A number of spherical lens variants are well known, and some are briefly reviewed below.

A classic type of spherical lens is the Luneburg lens [12], which focuses an incident plane wave at a point on the lens surface. The spherical geometry allows for multiple beams to be produced from multiple feeds [14], and it is this property which is attractive for a HAP payload. The Luneburg lens requires a radial variation of dielectric constant:

\[
\varepsilon_r = 2 - \left(\frac{r}{R}\right)^2
\]

where \( r \) is radius within the lens, and \( R \) is the outer radius. Thus the dielectric constant varies from 1 at the lens edge to 2 at the centre and there is no reflection loss due to abrupt transitions. This leads to a very good aperture efficiency, but it somewhat problematic to manufacture. In practice such a lens is often fabricated from a set of concentric shells and this inevitably degrades the efficiency: a useful treatise is given by [15]. However, for limited bandwidth applications a two-shell design can give almost equivalent performance [16].

A single shell spherical lens has no single focus, but exhibits a paraxial focus when a small proportion of the lens is illuminated. The paraxial focus may lie inside or outside the lens outer radius. A low dielectric spherical lens can give good performance - e.g. [17] reports on a Teflon lens (\( \varepsilon_r = 2.08 \)) for wide-scanning automotive radar at 77 GHz - and is of course much more simple to manufacture than a multi-shell lens.

Using the Luneburg lens aperture distribution given by [12], where the primary feed pattern is also taken into account, the far field radiation pattern can be computed. The geometry is illustrated in Fig. 1 for a generalised lens where the focus may lie inside or outside the lens radius \( a \). However, for the remainder of the analysis we will assume a conventional Luneburg lens where \( f = a \).

![Fig. 1. Morgan's geometry [12] for generalised Luneburg lens showing aperture plane](image)

A convenient model for the primary feed power pattern \( P_{PP} \) as a function of angle \( \theta' \) is:

\[
P_{PP} = \cos[\theta']^n
\]

where \( \theta' = \arcsin\frac{r}{f} \)

then from (2) and [12] we obtain for the radial power distribution \( P(r) \) in the aperture plane:

\[
P(r) = \frac{\cos\left[\arcsin\left(\frac{r}{f}\right)\right]^n}{f^2 \sqrt{f^2 - r^2}}
\]
The power pattern $P(\theta)$ of (4) may be derived in general by the transform to the far field. However, for $n=1$ the $r$ dependence of $P(r)$ disappears and we have a uniform aperture. This of course yields the highest aperture directivity for a given diameter $2r$ and also serves as a useful shortcut for deriving the radiation patterns for each HAP payload antenna because the standard formula for a uniform aperture may be used [13]. Where a more severe primary feed roll-off is applied i.e. by increasing $n$, the lens aperture plane experiences a tapered distribution, leading to lower sidelobes but also a reduced directivity; in such a case the lens diameter must be increased to recover the required half power beamwidth (HPBW) for each cell and this is later explored in detail. Radiation patterns from (3), for a lens diameter of 10 wavelengths, are shown in Fig. 2. When the $n=1$ model is used to generate CIR in a multi-cell network as described below, this type of radiation pattern gives the lowest CIR values due to the high sidelobe levels.

**POWER CONTOURS AND CO-CHANNEL INTERFERENCE.**

In contrast to earlier studies where each antenna beam is optimised, the use of multiple-beam spherical lens antennas leads to each beam of a given cluster having the same beamwidth and also having circular symmetry. Taking the layout of 121 cells shown in Fig. 3, where cells are arranged in 6 concentric hexagonal rings, we can specify an antenna beamwidth associated with each ring based on the mean required beamwidth. This is a modification of the methodology of [5] and leads to non-circular cellular power footprints but allows the use of a single spherical lens aperture for each ring and thus a much more compact antenna payload. Elements of such a payload are illustrated in Fig. 4, which shows how the innermost group of cells are served by the smallest spherical lens with a cluster of appropriately spaced feeds, and the outermost group of lens are similarly served by the largest spherical lens. The lens diameters indicated are approximations based on uniform aperture illumination and a 28 GHz carrier frequency and thus serve as a benchmark estimate for minimum antenna dimensions at this frequency.
Fig. 5 shows power contours for a co-channel cell group where case (a) is for dedicated beams which are asymmetric, and case (b) is for circular symmetric beams which could be generated using multi-beam spherical lenses. The difference in these two methods is apparent when we observe how the circular beams lead to non-circular cell power contours. The HAP payload in the former case comprises 121 individual antennas, while in the latter case it comprises 6 spherical lenses (one for each hexagonal ring), plus a single antenna for the centre cell. The lenses require multiple feeds, these typically being waveguide horns. In the outer ring the beam pointing angle is typically 56° from vertical and the azimuth spacing is typically 10.1°. As lens diameter for 28 GHz would be not less than 100 mm, this leads to a feedhorn spacing of about 15 mm (Fig. 4) which is adequate for a small horn fed by circular waveguide or a dielectric-loaded horn.

![Power contours for dedicated beams and circular beams](image)

(a) dedicated beams.  
(b) Multiple beam spherical lenses

Fig. 5. Power contours for 1 of 3 channels (contour spacing is 1.5 dB).

The co-channel CIR values associated with the two different payload types of Fig. 5 (a) and (b) are presented as cumulative distributions in Fig. 6. Comparing the dedicated, asymmetric beam case with the multi-beam spherical lens case the CIR difference is typically between 5 - 7 dB across the coverage area. This trend is expected because the dedicated beams model uses a flat sidelobe floor at -40 dB, while the circular beams model uses the theoretical sidelobe structure for uniformly illuminated circular apertures (Fig. 2) whose mean relative level is somewhat greater than -40 dB. Hence we are able to present a comparison of CIR performance for (i) an established payload model using dedicated beams and (ii) a more compact, less massive payload based on a group of minimum size spherical lenses. The term “minimum size” refers to the assumption that the aperture is uniformly illuminated and is thus, for each cell group, the smallest lens aperture which can be used to generate the required beamwidths.

![CIR coverage distribution](image)

Fig. 6. CIR coverage distribution.
REDUCING THE SPHERICAL LENS SIDELOBE LEVEL.

In the multi-beam spherical lens case the CIR values can be improved by using bigger lenses with tapered aperture distributions since this leads to lower sidelobe levels. In this section a recipe is sought from which we may estimate the dimensions of a multi-beam spherical lens payload which would deliver CIR values equivalent to the dedicated beam payload. A tapered aperture distribution implies that a directive primary feed is used. This is also equivalent to \( n > 1 \) in (4). The effect of the widening of the secondary beam is shown in Figure 2.

If the primary feed pattern is given by (3) then, for a given primary feed HPBW \( \theta_p \), since \( \cos(0.5 \theta_p)^n = \frac{1}{2} \) (5) we have

\[
n = \log_{\cos(0.5 \theta_p)} \left( \frac{1}{2} \right)
\]  

(6)

and thus the primary feed pattern is described by a single parameter \( n \) which is derived from its HPBW. We may then relate the aperture field distribution to the far field by employing the integral:

\[
E(\theta) = \int_{r=0}^{\infty} A(r) r J_0(k a \sin(\theta) r) \, dr
\]

(7)

where \( A(r) \) is the radial aperture field distribution which is given by \( A(r) = \sqrt{P(r)} \) from (4). We may then evaluate (7) for a given combination of aperture radius \( a \) and primary feed HPBW so as to derive the HPBW of the secondary field.

One approach is to plot the main lobe patterns and in each case search for the half-power points using a computer algorithm. This procedure lead to the results summarised in Fig. 7 where the primary feed HPBWs chosen are 30°, 45°, 60° and 90° and these parameters give rise to the discrete data points which are shown.

In Fig. 7(a) can be seen the expected trend that a larger aperture gives rise to a narrower secondary beam, while a narrower primary feed beam gives rise to a wider secondary beam. The latter effect is also associated with a reduction in sidelobe levels. Fig. 7(b) shows the increase in secondary beamwidth, relative to the uniform aperture \( (n=1) \), as a function of aperture radius \( a \) and for the four cases of primary HPBW chosen. Here we find a very useful result: for a given primary feed, the widening of the secondary beam is by an approximately constant factor for the range of aperture radii chosen.

Considering the HAP antenna payload, we now have a recipe for estimating the amount by which the antenna beamwidths would need to be increased when the lens aperture experiences an amplitude taper brought about by a generic primary feed. The beam widening must be countered by a commensurate and linear increase in the lens radius so as to recover the beamwidth which is required by each cell group. The calculated scaling factors are listed in Table 1 and summarised as an extrapolated function of primary feed HPBW in Fig. 8.

\[\text{(a) Secondary HPBW versus primary HPBW for radii in wavelengths as labelled}\]

\[\text{(b) Beamwidth widening factor compared to uniform aperture.}\]

Fig. 7. Relationship between primary feed and secondary HPBW for Luneburg lens.
Table 1. Summary of Luneburg lens beam widening and sidelobe levels for various primary feed beamwidths.

<table>
<thead>
<tr>
<th>primary feed HPBW (°)</th>
<th>beam scaling factor</th>
<th>first sidelobe level relative to peak gain (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>uniform aperture</td>
<td>1</td>
<td>-17</td>
</tr>
<tr>
<td>90</td>
<td>1.06</td>
<td>-19</td>
</tr>
<tr>
<td>60</td>
<td>1.22</td>
<td>-24</td>
</tr>
<tr>
<td>45</td>
<td>1.43</td>
<td>-30</td>
</tr>
<tr>
<td>30</td>
<td>1.87</td>
<td>-44</td>
</tr>
</tbody>
</table>

The derivation of CIR for the co-channel cell group now requires (7) to be evaluated so as to derive the power footprints of each cell. (The integration involved in (7) is much more computationally intensive than the previously reported radiation pattern models and thus leads to much longer computer run times.) Some results for computed CIR distributions are shown in Fig. 9 for multi-beam antennas using various primary feed beamwidths and where the aperture diameter is scaled accordingly so as to maintain the required secondary beamwidth for each cell group. In Fig. 9 we see the expected trend that CIR levels increase as the diameter of each lens is increased while at the same time illuminating the lens using primary feeds of reduced beamwidth. The case for the 60° primary feed / spherical lens payload shows approximately equivalent performance to the dedicated beam model, and from this may be drawn interesting conclusions about the relative masses of the two payload types.

![Fig. 8. Increase in secondary beamwidth, compared to that of uniform aperture, as a function of primary beamwidth.](image)

![Fig. 9. CIR distributions for dedicated and multi-beam (spherical lens) payloads.](image)

**RELATIVE PAYLOAD MASS.**

To close the analysis we present an estimate of the total mass for the HAP antenna payloads which have been discussed. For the dedicated beam payload, a working estimate for total lens mass can be derived as follows: Using the mass of our experimental asymmetric beam lens antenna [11] as a benchmark, and noting that, to a good approximation, there are straightforward relationships between HPBW, mass, and lens diameter D:

\[
\text{HPBW} \propto \frac{1}{D} \quad \text{and} \quad \text{mass} \propto D^3
\]

hence by scaling to the 466 g. mass of the experimental polyethylene lens whose HPBW in the narrow plane\(^1\) is 5° we obtain an estimate for each lens mass in the dedicated beam payload:

\[
\text{mass} = 466 \left( \frac{5}{\text{HPBW}_{\text{degrees}}} \right)^3 \quad \text{(grammes)}
\]

Using the iterative method of [5] to derive HPBW for each cell in the 121 cell network yields a total lens mass of 28.8 kg. The figure is somewhat tentative, and has been derived for a HAP at a height of 20 km above a 60 km diameter service area. Clearly, if these parameters are altered, the antenna beamwidths (which are tailored to each cell subtended

\(^1\) The experimental asymmetric beam lens antenna has a circular cross section whose diameter is dictated by the narrow (elevation) beamwidth of the cell. The beamwidth in the azimuth direction is broadened by modifying the lens surface [11].
angle) are altered and thus the lens masses also change. The mass of the primary feeds has not been considered in this analysis.

The estimate for the mass of the multi-beam spherical lens payload can be derived from the diameter of each lens and (assuming a single shell lens design) the material specific gravity. Again assuming a polyethylene material, whose density is 947 kg/m³, and uniformly illuminated apertures for the minimum size payload case, a mass of 1.93 kg is derived for the group of 6 spherical lenses. The figure would be reduced for multi-shell or Luneburg type lenses. Again, the mass of the primary feeds has not been considered, as it is possible that these could be very similar for the two payload cases and their number would also be the same.

A comparison of the dedicated beam payload, assuming -40 dB sidelobe floors, and multi-beam spherical lenses are summarised in Table 2. The CIR levels offered by the dedicated beam payload may, to a good approximation, be replicated by using the spherical lens payload with 60° beamwidth primary feeds. The lens diameters must also be increased by a factor of 1.22 from which we observe that the increase of lens mass is 1.22³ = 1.82. It is stressed that the masses presented are estimates based on extrapolation from those of representative components.

<table>
<thead>
<tr>
<th>Number of primary feeds</th>
<th>Number of lenses</th>
<th>Estimate of total lens mass (kg)</th>
<th>CIR (dB) (minimum / maximum)</th>
</tr>
</thead>
<tbody>
<tr>
<td>dedicated beams</td>
<td>121</td>
<td>121</td>
<td>28.8</td>
</tr>
<tr>
<td>multi-beam spherical lenses:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i) minimum aperture size</td>
<td>121</td>
<td>6</td>
<td>1.93</td>
</tr>
<tr>
<td>(ii) scaled aperture with 60° primary feeds</td>
<td>121</td>
<td>6</td>
<td>3.50</td>
</tr>
</tbody>
</table>

CONCLUSIONS

HAPs offer an effective way of exploiting mm-wave spectrum by supporting multi-cell architectures. The viability of such systems is largely determined by the overall system data capacity, which is in turn governed by the properties of the antenna payload which serves the cells on the ground. In previous studies, “ideal” antenna beams have been used to model carrier-to-interference ratio. In such cases, dedicated aperture antennas such as lens antennas may be used to produce the required beam shapes which are in general asymmetric. This approach leads to one antenna for each cell and thus a bulky payload.

Where multi-beam antennas are used the mass and volume of the HAP payload can be much reduced. A convenient approach is to employ spherical lens antennas using multiple feeds. This work has considered the properties of such antennas and shown how for a 121 cell network the payload may be reduced to just 6 multi-beam antennas. Disadvantages of this antenna type is that radiation patterns are in general of circular cross-section, which leads to non-circular cell footprints, and a compromise beamwidth must be chosen for a given cell group.

The resulting CIR levels have been computed for spherical lens antennas where the aperture distribution is uniform, this being the smallest antenna but with the highest sidelobes. The technique was then extended to consider the effect of the primary feed beamwidth, since this may be chosen to taper the aperture distribution and thus reduce sidelobe levels, but must be accompanied by an increase in lens diameter so as to maintain the required directivity for the cell group. A generalised recipe was found whereby the aperture scaling term was derived for a range of primary feed beamwidths. This allowed direct comparison of the relative size of the antenna payload for given CIR levels. The trade-off is one where the smallest antennas have the highest sidelobes and hence lowest CIR, while increasing the antenna size leads to increased CIR levels.

The benchmark payload, using 121 dedicated antennas where each is assumed to have a mean sidelobe floor at -40 dB, offers CIR levels between 12 dB and 18 dB and was shown to have a total dielectric lens mass estimated at 28.8 kg. Compared to this, a payload of 6 multi-beam spherical lenses of minimum possible size would offer a mass reduction of at least 93 % (assuming single shell, not Luneburg lens) but a CIR degradation of around 6 dB. It was shown that the
CIR levels offered by the dedicated antenna payload may also be obtained by the multi-beam spherical lens payload if each lens aperture diameter is increased by a factor of 1.22 in conjunction with primary feeds of 60° half-power beamwidth. The total lens mass in this case would be approximately 88% less than that of the benchmark payload.

While operational HAP communications services have yet to be deployed in practice, it has been shown that a group of multi-beam spherical lenses could offer a very practical and compact antenna payload. This would support multi-cell architectures with adequate levels of co-channel interference and with a very considerable mass saving compared to a payload of a type which uses one lens antenna for each cell.

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REFERENCES.